The Islamia University Of Bahawalpur,
Department of Computer Science & IT
Bahawalnagar Campus
Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Topic: Practice Questions (Polynomial Approximations)

Solutions.

lect-11

Practise Question:-

16-3-17 (64)

No.3 Find the natural cubic spline which interpolate the data. Also find the $f(6)$?

x	2	4	5	7
$f(x)$	1.34	1.84	1.12	0.02

Q: Find the best fit straight line to the following experimental data.

x	0.00	1.00	2.00	3.00	4.00
$f(x)$	1.00	3.85	6.50	9.35	12.05

Solution:

$h_1 = 2$; $h_2 = 1$; $h_3 = 2$

For a natural cubic spline we required $S''(x_1)$ and $S''(x_4)$ are zero.

The value of S'' at other data points are found from the system of equations

$$\underbrace{h_1 S''(x_1)}_{=0} + 2(h_1+h_2) S''(x_2) + h_2 S''(x_3) = r_2$$

$$2(2+1) S''(x_2) + 1\, S''(x_3) = 6\left(\frac{f_3-f_2}{h_2} - \frac{f_2-f_1}{h_1}\right)$$

$$= 6\left(\frac{1.12-1.84}{1} - \frac{1.84-1.34}{1}\right)$$

$$6 S''(x_2) + S''(x_3) = -5.82 \quad \text{--- (i)}$$

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Now 2nd Eq.

$$h_2 S''(x_2) + 2(h_2+h_3) S''(x_3) + h_3 \underbrace{S''(x_4)}_{=0} = r_3$$

$$1 S''(x_2) + 2(1+2) S''(x_3) = 6\left(\frac{f_4-f_3}{h_3} - \frac{f_3-f_2}{h_2}\right)$$

$$= 6\left(\frac{0.02-1.12}{2} - \frac{1.12-1.84}{1}\right)$$

$$S''(x_2) + 6 S''(x_3) = 1.02 \quad \text{—(ii)}$$

Multiplying by '6' above eq. we get

$$6 S''(x_2) + 36 S''(x_3) = 6.12 \quad \text{—(iii)}$$

$$\cancel{\pm 6 S''(x_2)} \pm S''(x_3) = \mp 5.82$$

$$35 S''(x_3) = 11.94$$

$$S''(x_3) = \frac{11.94}{35}$$

$$\boxed{S''(x_3) = 0.341143}$$

By putting $S''(x_3)$ into eq (i)

$$6 S''(x_2) + 0.341143 = -5.82$$

$$= -5.82 - 0.341143$$

$$S''(x_2) = -6.161143/6$$

$$\boxed{S''(x_2) = 1.02686}$$

As we have to find f(6) which lies in lies in $(5 < x < 7)$
So first we find the spline for the interval

$$S(x) = a_3(x-x_3)^3 + b_3(x-x_3)^2 + c_3(x-x_3) + d_3 \quad \text{—(iv)}$$

Since (66)

$$a_3 = \frac{S''(x_4) - S''(x_3)}{6h_3}, \quad S''(x_4) = 0$$

$$a_3 = \frac{-0.341143}{6\times 2} = \frac{-0.341143}{12} = -0.02843$$

$$\boxed{a_3 = -0.02843}$$

$$b_3 = \frac{S''(x_3)}{2} = \frac{0.341143}{2}$$

$$\boxed{b_3 = 0.170571}$$

$$c_3 = \frac{f_4 - f_3}{h_3} - \left(\frac{S''(x_4) + 2S''(x_3)}{6}\right)h_3$$

$$= \frac{0.02 - 1.12}{2} - \left(\frac{2(0.341143)}{6}\right)2$$

$$\boxed{c_3 = -0.777429}$$

$$d_4 = f_4 = 1.12$$

Substitute these values in eq (iv)

$$S(x) = 0.02843(x-5)^3 + 0.170571(x-5)^2 - 0.777429(x-5) + 1.12 \iff (5 < x < 7$$

For $f(6) = ?$

$x = 6$; use above spline $S(x)$

$$S(6) = -0.02843(6-5)^3 + 0.170571(6-5)^2 - 0.777429(6-5) + 1.12$$

$$S(6) = -0.02843 + 0.170571 - 0.777429 + 1.12$$

$$S(6) = -0.805859 + 1.290571 = 0.484712$$

(67)

As spline has a nice property
$\rightarrow S(x) = f(x)$
So $S(6) = f(6) = 0.484712$

$\Rightarrow$ Find $f(3)$ for this data

$f(3)$ lies in interval $(2 < x < 4)$

Key-points.
$\rightarrow S(x) = a_1(x-x_1)^3 + b_1(x-x_1)^2 + c_1(x-x_1) + d_1$

$\rightarrow$ Find a_1, b_1, c_1, d_1
$\rightarrow$ we already ~~find~~ have values of
$\underbrace{S''(x_1)}_{=0}$; $\underbrace{S''(x_2)}_{=1.02686}$; $\underbrace{S''(x_3)}_{=0.341143}$; $\underbrace{S''(x_4)}_{=0}$

Natural splines

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Q. Find the best fit straight line to the following experimental data

x	0.00	1.00	2.00	3.00	4.00
$f(x)$	1.00	3.85	6.50	9.35	12.05

Sol

x_n	f_n	x_n^2	$x_n f_n$
10.00	32.75	30.00	93.10

$$\sum_1^n 1 = 5$$

$$c\sum_1^n 1 + m\sum_1^n x_n = \sum_1^n f_n \quad \text{—(i)}$$

$$c\sum_1^n x_n + m\sum_1^n x_n^2 = \sum_1^n x_n f_n \quad \text{—(ii)}$$

$$5c + 10m = 32.85 \quad \text{— (A)}$$

$$10c + 30m = 93.10 \quad \text{— (B)}$$

$$y = \underbrace{1.03}_{=c} + \underbrace{2.76}_{=m} x$$

(69)

7-3-17
FRIDAY

An experimenter is carried out and the data obtained are as follows.

x_u	0.2	0.3	0.5	0.9
f_u	5.54	4.02	3.11	2.16

Find the least square best fit straight line $y = mx + c$

Sol.

x_u	f_u	x_u^2	$x_u f_u$
0.2	5.54	0.04	1.108
0.3	4.02	0.09	1.206
0.5	3.11	0.25	1.555
0.9	2.16	0.81	1.944
1.9	14.83	1.19	5.813

The pair of equation for 'm' & 'c'

$$c\sum_1^n 1 + m\sum_1^n x_u = \sum_1^n f_u$$

$$4c + 1.9 = 14.83 \quad \text{---(i)}$$

And $$c\sum_1^n x_u + m\sum_1^n x_u^2 = \sum_1^n x_u f_u$$

$$1.9c + 1.19m = 5.813 \quad \text{---(ii)}$$

7c

By solving the eq (i) & (ii)
we get

$$m = -4.28$$

and $c = 5.74$

So

$$y = mx + c$$

$$\boxed{y = -4.28x + 5.74}$$

Q No. 2

Use the polynomial approxi-
-mation to estimate $f(22)$

x	$\underset{10}{x_1}$	$\underset{15}{x_2}$	$\underset{20}{x_3}$	$\underset{25}{x_4}$
$f(x)$	9.23	8.41	7.12	4.13

Sol. we apply the exact data
method for $f(22)$

First we find lagrange polynomial
at $x = 22$

So we consider

$$L_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$L_1(22) = \frac{(22-15)(22-20)(22-25)}{(10-15)(10-20)(10-25)} = 0.056.$$

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$$L_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$L_2(22) = \frac{(22-10)(22-20)(22-25)}{(15-10)(15-20)(15-25)} = -0.288$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$L_3(22) = \frac{(22-10)(22-15)(22-25)}{(20-10)(20-15)(20-25)} = 1.008$$

Similarly $L_4(22) = 0.224$

Now our required interpolated polynomial, at $x = 22$ is

$$p(x) = f_1 L_1(x) + f_2 L_2(x) + f_3 L_3(x) + f_4 L_4(x)$$

i.e. $p(22) = f_1 L_1(22) + f_2 L_2(22) + f_3 L_3(22) + f_4 L_4(22)$

$$p(22) = 9.23 \times 0.056 + 8.41 \times -0.288 + 7.12 \times 1.008 + 4.13 \times 0.224$$

$$p(22) = 6.197$$

As $p(22) = f(22) = 6.20$

which is appropriated value of $f(x)$

END–lec

Best of Good luck